PROBLEM OF HEAT-FLUX SCREENING IN THE RADIATIVE–CONVECTIVE HEATING OF BODIES OVER WHICH A HIGH-ENTHALPY GAS FLOWS

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The results are presented on a calculated theoretical investigation of the radiative–convective heat exchange and the screening properties of the gaseous products of the ablation of heat-shield materials in hypersonic flow over the front surface of a smooth blunt body.

The branches of radiative gasdynamics (RGD) connected with the study and grounding of the physical models of flow of high-temperature gases, the development and perfection of numerical methods of multidimensional equations of motion of a radiating gas, and the theoretical and experimental study of the optical and thermophysical properties of gases and heat-shield materials (HSM) have undergone very fruitful development by many investigators in the last decade. A number of important results having great scientific and practical importance have been obtained.

It should be noted, however, that despite the clear achievements, the quantitative data on the radiative and convective heat fluxes to different sections of the structural elements of power devices over which a high-enthalpy gas flows were obtained by calculation mainly in the absence of the injection of coolant. Concerning the determination of the level of heating of bodies with allowance for radiative screening by the injection of optically active admixtures, it can be stated that up to now only individual results exist, allowing one to judge more the possibilities of the methods used and not the laws of transformation of heat fluxes in the boundary layer.

It should be emphasized that the determination of the general laws reflecting sufficiently fully the connection between the complicated hydrodynamic and physicochemical processes forming the thermal regime at a surface over which flow occurs produces a circle of problems, the solution of which is important from the aspect of the perfection of existing engineering methods of calculating heat-shield systems and the creation of new ones, the choice of which determines to a considerable extent not only the efficiency but also the reliability of operation of an entire device as a whole.

Those few results having the character of universal functions were obtained for the vicinity of the stagnation point [1-4] on the basis of an analysis of a limited number of calculated data, leaving open the question of the limits of applicability of these functions and the possibility of their extrapolation to other conditions.

The situation is complicated to a considerable extent by the limited possibiliites of the experimental study of combined radiative–convective heat exchange.

A solution of the problem of the screening of powerful radiant fluxes by the injection of coolant under conditions of intense radiative–convective interaction is now possible, in our view, on the basis of:

1) the creation of an efficient physicomathematical apparatus and the use of it to carry out a comprehensive calculated-parametric investigation, including a study of the influence on the screening properties of the existing uncertainties in the optical and thermophysical properties of gases and materials;

2) an analysis of the physical mechanisms of transformation of selective emission in a gas nonuniform in temperature and chemical composition and clarification of the general laws determining the relationship between the basic processes connected with radiative heat exchange and the screening mechanisms;

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3) the creation of new experimental stationary devices allowing one to reproduce not only the absolute values of the convective and radiant heat fluxes but also their spectral composition, and the development of methods for the correct conversion of test data to nature.

In the report we give some results along the first two directions, obtained in recent years at the Moscow Physics and Technology Institute, in application to the problem of the heating and destruction of a body braking in the dense layers of a planetary atmosphere.

One of the main questions arising in the formation of a calculated—theoretical study of the problem is that of the adequacy of the initial physical model to the nature of the phenomenon under investigation. Here we have in mind the fact that, despite the possibility in principle of a sufficiently full description, within the framework of modern theoretical concepts, of the complicated flow pattern of a high-temperature, chemically reactive, radiating gas mixture around a body being destroyed under the action of various heat-transfer mechanisms, the creation of a practical instrument for numerical investigation can be based only on a physically well-grounded simplification of the description of the processes playing an important role, on the one hand, and the allowance for which is associated with the need to use the most cumbersome calculation procedures, on the other.

From the point of view of the problem of screening of radiant fluxes under investigation the most important among these processes are the effects of molecular (laminar or turbulent) transfer in the multicomponent, chemically reactive boundary layer and radiant heat exchange in the gas in the presence of pronounced gradients of temperature and of the concentrations of the chemical components. In addition, the multidimensional character of the flow near the front surface of a specific body introduces considerable complication.

1. The overwhelming majority of the quantitative data on the characteristics of the radiative heating of bodies over which a hypersonic stream of viscous, thermally conducting, multicomponent gas flows have been obtained on the basis of calculations made using the model of "binary" diffusion. Among the few reports in which an attempt was made to allow for the effects of multicomponent diffusion (MCD) [5, 6] should be mentioned. The idea of "bifuraction" was used in [5] to describe the diffusion processes, while an approach based on the introduction of group concentrations and diffusional fluxes and the solution of the Stefan—Maxwell equations for these functions was used in [6]. However, the small set of calculated variants and the absence of a comparative analysis in these reports prevent one from judging the size of the error introduced into the determination of the characteristics of radiative heat exchange by the use of the approximate means of describing diffusion.

In the present work we compared the results of calculations of the integral and spectral radiant fluxes emitted by shock layers of various chemical compositions under the conditions of local thermodynamic equilibrium (LTE), both in the case of an impermable surface and when destruction of the heat-shield coating (HSC) is present. The data were obtained in a wide range of the parameters of flow over a spherically blunt body using the model of "binary" diffusion, on the one hand, and the method of effective coefficients [8],* on the other. For simplicity, we considered cases of a two-element composition in the shock layer. For example, if pure graphite was taken as the injected gas, then the atmospheres of the Earth, Venus, and Jupiter were modeled by pure nitrogen, carbon dioxide, and hydrogen, respectively.

An analysis of the results showed that with a fixed injection velocity the largest difference in the values of the radiant fluxes q_w to the surface in the physical models being compared occurs in the cases when the optical properties of the injected mixture differ markedly from the properties of the atmospheric gas (the injection of graphite vapor into a hyrogen shock layer).

The distributions across the shock layer of the temperature T, the radiative fluxes q, and the graphite concentration C are presented in Fig. 1 as an example. A comparison of curves 1 and 3 shows that the maximum discrepancy in the values of q calculated "exactly" and on the basis of model concepts about the character of the diffusion is observed at the surface and comprises about 30% in the given variant. In the case of a more correct choice of the coefficient D_{12} (curve 2) the size of the discrepancy does not exceed 15%. The layer is more transparent optically with MCD than with "binary" diffusion.

A decrease in either of the factors determining the role of radiative heat exchange in

*Later the results obtained by this method are called "exact."

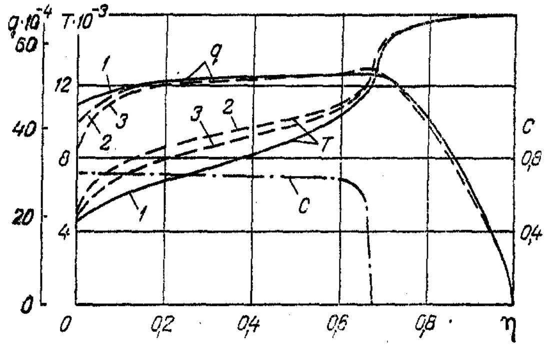

Fig. 1. Distributions of temperature T (°K), radiative flux q (kW/m^2) toward the body, and concentration C of the element graphite at the critical point of a hydrogen shock layer ($\eta = 0$ is the body and $\eta = 1$ is the shock wave) (parameters of flow over the body: $P_s = 10$ atm, $T_s = 16{,}000$°K, $T_w = 4500$°K, R = 0.5 m, $f_w = 0.1$, $\varepsilon_\lambda = 1$, injected mixture 70% C + 30% H_2): 1) calculation with allowance for the inequality of D_{ij} by the method of [8]; 3) "binary" diffusion [$D_{ij} = D_{12}$ ($C_2 - H_2$)]; 2) "binary" diffusion ($D_{ij} = D_{12}$, where D_{12} is calculated by the method of [16]).

the shock layer (T_s, P_s), the size of the body, or f_w results in lessening of the indicated difference in the values of q calculated within the framework of the approximate description of diffusion as compared with "exact" data. The influence of MCD effects on radiative heating also proves unimportant in practice if one considers the hypersonic flow of air or CO_2 over a blunt body covered by an HSM based on graphite.

The physical reason for the relatively weak influence of separation of chemical elements (due to the inequality of D_{ij}) on the radiative fluxes to the body integrated over the spectrum is that the optical depths, in the spectral interval in which most of the radiant energy is transferred, of the regions of flow with the maximum gradients, where this separation is manifested to the greatest extent, practically always prove to be considerably less than unity. At the same time, right near the surface, where the optically active molecules included in the composition of the HSM ablation products (such as CO, CN, C_2, and C_3) are present in appreciable quantities, there is practically no difference between the "binary" and MCD profiles of C.

Thus, on the basis of the above analysis we can formulate an important conclusion, permitting a well-founded simplification of the physical model of the flow of a viscous, heat-conducting, selectively radiating gas. In a wide range of conditions of flow over a body being destroyed under the action of combined radiative and convective heating by atmospheric gases of various chemical compositions, the use of the model of "binary" diffusion to describe transfer phenomena introduces an insignificant error into the value of q_w in comparison with the MCD values.

2. In a numerical solution of the equations of conservation of the energy of a radiating gas in the prsence of variable fields of T, P, and C most of the time is spent on determining the radiative fluxes q integrated over the spectrum and their derivatives.

In [9] a new approach was proposed, permitting the efficient determination of these characteristics of the radiation field in a plane-parallel gas layer in a state of LTE with arbitrary T and C gradients. The main idea of the method consists in the following. In a nonisothermal gas layer of variable elemental composition divided into N homogeneous layers the radiant flux q at an arbitrary point i of the coordinate grid is determined from the equation

$$q_i = q_i^0 + \sum_{k=0}^{N_e-1}\sum_{p=0}^{N} A_{ikp}^0 \delta z_{kp} + \frac{1}{2}\sum_{k=0}^{N_e-1}\sum_{m=0}^{N_e-1}\sum_{p=0}^{N}\sum_{q=0}^{N} B_{ikpmq}^0 \delta z_{kp}\delta z_{mq} + \dots, \quad (1)$$

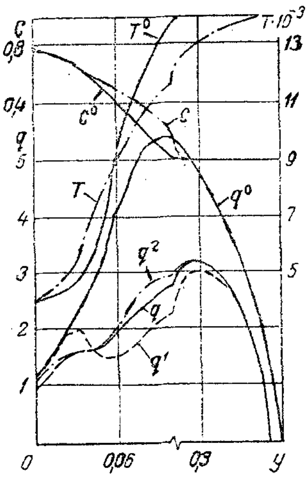

Fig. 2. An example of the results of a calculation of the radiant fluxes q (kW/m²) in an air shock layer near a graphite body. Flow parameters: P_s = 1 atm, T_s = 14,000°K, T_w = 4100°K, R = 1 m, f_w = 0.05, ε_λ = 1; T) temperature (°K); y) normal coordinate (cm).

where the vector q_i° and the matrices A_{ikp}° and B_{ikpmq}° are calculated on a reference profile of the generalized variable z_{ip}° (T and C profiles) from the equations

$$q_i^0 = \int\limits_\lambda\int\limits_\Omega I_i^0(\lambda,\ \Omega)\cos(n,\ \Omega)\,d\lambda d\Omega;\quad A_{ikp}^0 = \int\limits_\lambda\int\limits_\Omega\left[\frac{\partial I_i(\lambda,\ \Omega)}{\partial z_{kp}}\right]^0\cos(n,\ \Omega)\,d\lambda d\Omega;$$
$$B_{ikpmq}^0 = \int\limits_\lambda\int\limits_\Omega\left[\frac{\partial^2 I_i(\lambda,\ \Omega)}{\partial z_{kp}\partial z_{mq}}\right]\cos(n,\ \Omega)\,d\lambda d\Omega. \quad (2)$$

The fact that the integration over λ and Ω in (2) is carried out on reference profiles of temperature (T°) and of the concentrations of the chemical elements (C°), i.e., those known in advance, permits a considerable speedup in the calculation of the characteristics of the radiaton field.

The integrands in (2) are found by direct differentiation with respect to z_{ip} of the solution of the monochromatic equation of radiative transfer in each of the elementary layers [9]. It should be noted that Eq. (1) is simplified considerably in the case of binary diffusion (N_e = 2). Methodological calculations made for fixed profiles modeling the T and C distributions occurring in a shock layer near the surface of a body over which hypersonic streams of various chemical compositions flow showed the high efficiency of the proposed method. It was established, in particular, that for many cases of practical importance one can be confined to no more than two terms in the expansion (1). In addition, an analysis of the results showed that, with the appropriate choice of T°, an accuracy acceptable for practice (∿ 10%) is provided by allowance for only the diagonal terms in the matrix B_{ikpmq}° of second derivatives. This permitted a reduction by more than an order of magnitude in the requirement for computer memory.

Calculations of radiative heat exchange in a hypersonic shock layer near a blunt body of graphite were made along with the model calculations using the proposed method. The complete gasdynamic problem of flow and heat exchange in a chemically equilibrium shock layer was solved in these calculations. The T and C distributions obtained in each iteration were used as the working profiles in calculating the radiation field from equations of the type (1). In calculating the coefficients (2) to T° and C° we allowed for all the most important mechanisms of radiant-energy transfer, including the lines of atoms and ions, in the region of the spectrum from λ = 0.05 to λ = 1.5 µm. Distributions obtained from the solution of the gasdynamic problem without allowance for radiation were chosen for T° and C°, as a rule. In this case it turned out that in a wide range of the determining parameters (T_s, P_s, and R) the temperature factor introduces the main error in the calculation of the profiles, whereas the quantity q is more conservative with respect to variations of C. An analysis showed that a deviation in the value of q within the shock layer calculated from Eq. (1) not exceeding 10% from the exact values found through a direct solution of the equation of radiant-energy transfer for the T and C profiles is assured by the allowance for only linear terms in δC and quadratic (diagonal) terms in δT in (1).

The results of a calculation of the radiant fluxes at the critical point of an air shock layer for the characteristic conditions of flow over a blunt body are presented in Fig. 2. From a comparison of the q and q^1 profiles it is seen that the use of only a linear approximation in Eq. (1) can actually result in considerable differences of not only a quantitative but also a qualitative character. At the same time, the q^2 profiles coincide with the exact results to within 10%. We note that, owing to radiative cooling, the thickness of a radiating shock layer is less than its thickness in the absence of emission. Therefore, the right-hand limits of the reference and working profiles differ in the figure.

3. The presently existing methods of numerical investigation of a radiating shock layer near the front surface of a blunt body, based on the concepts of a viscous [1, 4, 10] and an ideal [11-13] physical model of flow over a body, represent a formal uniting of the algorithms for solving the strictly gasdynamic (three-dimensional, in general) problem on determining the fields of velocity, pressure, and enthalpy and the shape and position of the shock wave and for calculating the selective-emission field and the characteristics of the destruction of the HSC. This predetermines a certain limitation on the possibilities for using such methods to carry out a calculated-parametric investigation. Approaches using a combined formulation [14] possess certain advantages from the point of view of increasing the economy of the calculation methods. Its essence consists in the separate solution of the equations of inviscid flow with allowance for emission near an impermeable body and the equations describing flow in the boundary layer and destruction of the HSC.

For the given initial shape of the body one solves the direct problem of the inviscid flow of a radiating gas. As a result, one finds the position and shape of the shock wave, as well as the distributions of temperature, pressure, tangential velocity, and the spectral characteristics of the radiation field along the front surface. These are used as the boundary conditions for the integration of the equations of a laminar or turbulent boundary layer. The values of the radiant and convective heat fluxes found through the solution allow one to determine the characteristics of the destruction of the HSM. Then one finds the new physical thickness of the boundary layer, i.e., one actually determines the new shape of the impermeable body, for which the inviscid problem is solved again. The procedure is repeated until convergence. In such an approach the simplification is achieved, first, through the fact that instead of the integration of complicated equations of the viscous and heat-conducting shock layer one solves the simpler Euler equation and boundary-layer equation. Second, to find the values of the radiant and convective heat fluxes and the characteristics of HSC ablation one can resort to the procedure of calculating the inviscid part a small number of times.

A considerable increase in the efficiency of the numerical algorithm is possible within the framework of the combined formulation. In [15] a new method of radiation disturbance (MRD) was proposed for calculating a radiating inviscid shock layer near the front surface of an impermeable body with an arbitrary shape. The idea of the method is based on the valid (within a wide range of the determining parameters), physically well-founded assumption that the influence of radiant-energy transfer on the field of dynamic flow characteristics (the velocity, the pressure, and the shape and local inclination of the shock wave) is negligibly small, and it is possible to allow for this influence on the fields of thermodynamic functions (enthalpy and density) and on the thickness of the shock layer within the framework of linear corrections to the undisturbed values. This approach is equivalent to a rigorous solution of the problem on the basis of the asymptotic expansion of all the unknown functions with respect to the radiative–convective parameter $\Gamma = 4q_{ad}/\rho_\infty U_\infty^3 (\Gamma < 1)$; here one is confined to zeroth-order terms for the dynamic variables and first-order terms for the thermodynamic variables. The fact that the linear equations of continuity and energy conservation obtained relative to the corrections contain as coefficients the functions calculated from a known solution for a nonradiating shock layer eliminates the need to make a gasdynamic calculation proper. The divergence of the radiant flux, appearing in the equation of energy conservation, was found by the method of nonlocal averaging presented above, and adiabatic distributions were also used as the reference temperature profiles. This

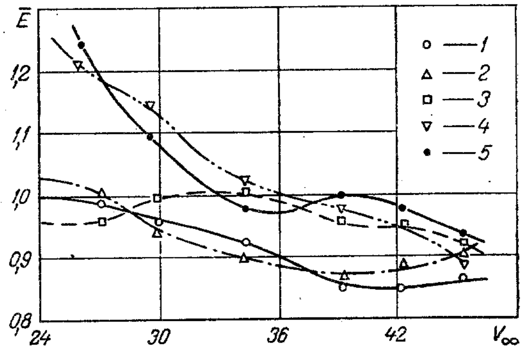

Fig. 3. Variation of the dimensionless coefficient of screening $\overline{E}(\overline{x}) = E(\overline{x})/E(0)$ of radiant flux along the braking trajectory of a spherically blunted cone ($R = 0.3$ m, $\Theta_c = 60°$, $R_m = 0.5$ m) in an atmosphere consisting of 85% H_2 + 15% He (by volume): 1) $\overline{x} = 0.5$; 2) 1.0; 3) 1.5; 4) 2.0; 5) 2.5. V_∞, km/sec.

made it possible to carry out integration over the spectrum and angles only once (in the first iteration). The good agreement between the results of [15] and the data of calculations obtained by other authors within the framework of a formulation using joint integration of the equations of gas dynamics and the equation of radiation transfer confirms the soundness of the initial assumptions, permitting a considerable increase in the speed of the entire algorithm without a loss of accuracy. For example, the characteristic time of a complete gasdynamic calculation of a radiating shock layer near the front surface of a sphere over which a hypersonic air stream flows is about 20 min (on a BÉSM-6 computer). At the same time, the MRD, when a solution for a nonradiating shock layer is present, assures a solution of the problem of radiative heating of a sphere (using a given spectral model of the optical properties of air) in 10 sec.

Using the method developed we made systematic calculations of the radiative—convective heat exchange and screening properties of the injected gas in the hypersonic flow over the front surface of smooth blunt bodies of different shapes by air and $CO_2 + N_2$ and H_2 + He mixtures.

An analysis of the results obtained allowed us to establish a number of laws connected with the screening of radiant fluxes integrated over the spectrum by the gaseous products of HSM ablation over a considerable part of the front surface of a body. It was established, in particular, that in a wide range of the geometrical characteristics and parameters of flow by a gas of a given elemental composition (ρ_∞, V_∞) over convex bodies (a sphere and a spherically blunted cone with $\theta > 40°$) covered by carbon-based HSM the coefficient of screening $E(\overline{x}) = q_w(\overline{x})/q_e(\overline{x})$ in the shock-layer cross section under consideration is connected with the coefficient of screening $E(0)$ at the leading critical point by the following approximate relation:

$$E(\overline{x}) = E(0)\, F(\overline{x},\ V_\infty). \tag{3}$$

The function $F(\overline{x}, V_\infty)$ has the simplest form in the case of laminar flow over a front surface having the shape of a blunt cone: $F(\overline{x}, V_\infty) \approx 1$.

Data illustrating this result are presented in Fig. 3. Pure graphite was considered as the HSM. Different values of the longitudinal coordinate $\overline{x}$ are marked by points. It is seen that the maximum difference of $E(\overline{x})$ from unity actually does not exceed 10% in the range of flow conditions in which the heating of the surface is determined mainly by radiation ($V_\infty \geqslant 30$ km/sec). Violation of this approximate self-similarity for flow regions remote from the critical point does not play a significant role at lower flow velocities, since heating of the surface by radiation is unimportant under these conditions.

Thus, the binary approximation can be used to describe the processes of diffusion in a wide range of the determining parameters in the construction of a calculated-theoretical model of the flow of a high-enthalpy, strongly radiating gas over a body; the appropriate diffusion coefficient must be determined from the condition that the binary Lewis number equals unity.

In problems connected with the need to calculate the radiation field in a nonisothermal gas with a variable elemental composition (in the LTE state) with a priori known ranges of variation of temperature $|\delta T|$ and of the concentration of the element $|\delta C|$ due to the influence of radiation, $|\delta T/T_{max}| \simeq |\delta C/C_{max}| \leqslant 0.25$, the use of the proposed method of integration over the spectrum lets one take into account, with a high accuracy and with the minimum expenditures of machine time, the radiative—convective interaction and to determine the radiant heat flux to the surface.

On the basis of the developed complex of efficient methods we made a calculated-parametric investigation of the radiative—convective heat exchange and screening properties of HSC destruction products on the front surface of a blunt body over which a hypersonic stream flows. It was established that in a wide range of conditions of flow over smooth blunt bodies the coefficient of screening in an arbitrary cross section of the shock layer is connected with its value at the critical point by a universal function dependent only on $\bar{x}$ and the velocity of motion of the body in the atmosphere.

NOTATION

T, temperature; P, pressure; C, weight concentration of an element; c_i weight concentration of the i-th component; q, radiative heat flux directed from the shock wave toward the body; R, radius of a sphere; R_m, radius of base of a cone; $f_w = (\rho v)_w/\rho_\infty V_\infty$, injection parameters; v, normal velocity; D_{ij}, binary diffusion coefficient; N, number of chemical components; N_e, number of chemical elements; $\bar{x} = x/R$, longitudinal coordinate measured along the front surface from the spread-out line; n, normal to the surface; η, Dorodnitsin variable; λ, wavelength; θ, half-angle of cone apex; ρ, density; ε, emissivity of surface; Ω, unit vector characterizing the direction of propagation of radiation. Indices: s, conditions behind the shock wave; e, at the outer limit of the boundary layer; w, at the wall; ∞, in the oncoming gas; λ, spectral characteristic.

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CONTROL OF THE TEMPERATURE REGIME IN A LAYER OF HEAT-CONDUCTING MATERIAL

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UDC 536.24.02

The article presents the solution of the problem of finding the optimum controlling heat flux on the boundary of a plane heat-conducting plate ensuring stabilization of the temperature in the specified section with known disturbing heat flux on the other boundary.

We are concerned with an infinite plane plate with thickness d, with the heat flux z(t) being specified on one of its boundaries. We have to find such a heat flux u(t) on the other boundary that in the specified section $x_0 \in [0, d]$ the regularity of change of temperature y(t) is ensured.

In dimensionless form the problem is described by the one-dimensional heat-conduction equation

$$\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial x^2} \tag{1}$$

with boundary conditions of the second kind

$$\left.\frac{\partial \Theta}{\partial x}\right|_{x=0} = z(t); \left.\frac{\partial \Theta}{\partial x}\right|_{x=\pi} = u(t),\ t \geqslant 0, \tag{2}$$

the initial condition

$$\Theta|_{t<0} = \Theta_0(x) \tag{3}$$

and the condition

$$\Theta_{x=x_0} = y(t),\ t \geqslant 0,\ x_0 \in [0,\ \pi]. \tag{4}$$

In a fairly similar statement Kuznetsov [1] investigated the problem of stabilization $\Theta(x_0, t)$ without disturbing effect z(t). Stated somewhat similarly, the authors of [2, 3]

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